WEEKLY TEST TYJ -1 TEST - 35 R
SOLUTION Date 12-01-2020

[PHYSICS]

1. **(a)** Time required for a point to move from maximum displacement to zero displacement is

$$t = \frac{T}{4} = \frac{1}{4n}$$

$$\Rightarrow \quad n = \frac{1}{4t} = \frac{1}{4 \times 0.170} = 1.47 \text{ Hz}$$

2. **(b)** Since the point $x = 0$ is a node and reflection is taking place from point $x = 0$. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\lambda/2$

So, if $y_{\text{incident}} = a\cos(kx - \omega t)$

$$\Rightarrow \quad y_{\text{reflected}} = a\cos(-kx - \omega t + \pi)$$

$$= -a\cos(\omega t + kx)$$

3. **(c)** Critical hearing frequency for a person is 20,000 Hz.

If a closed pipe vibration in N^{th} mode then frequency of vibration

$$n = \frac{(2N-1)v}{4l} = (2N-1)n_1$$

(where n_1 = fundamental frequency of vibration)

Hence 20,000 = $(2N - 1) \times 1500 \Rightarrow N = 7.1 \approx 7$

Maximum possible harmonics obtained are

1,3,5,7,9,11,13

Hence, man can hear up to 13^{th} harmonic

$= 7 - 1 = 6$

So, number of overtones heard = 6

4. **(d)** Path difference $(\Delta x) = 50 \text{ cm} = \frac{1}{2}\text{m}$

$\therefore$ Phase difference $\Delta\phi = \frac{2\pi}{\lambda}$

$$\Delta x \Rightarrow \phi = \frac{2\pi}{1} \times \frac{1}{2} = \pi$$

Total phase difference $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$$\Rightarrow \quad A = \sqrt{a^2 + a^2 + 2a^2 \cos\left(\frac{2\pi}{3}\right)}a$$

5. **(a)** In first overtone mode, $l = \frac{3\lambda}{4}$

$$\therefore \quad \frac{\lambda}{4} = \frac{l}{3} = \frac{1.2}{3} = 0.4 \text{ m}$$

Pressure variation will be maximum at displacement nodes, i.e., at 0.4 m from the open end.

6. **(c)** The frequency of A, $n_A = n + \frac{2}{100}n$

and the frequency of B, $n_B = n - \frac{3}{100}n$

According to question, $n_A - n_B = 6$

$$\therefore \quad \left(n + \frac{2}{100}n\right) - \left(n - \frac{3}{100}n\right) = 6$$

or $\frac{5}{100}n = 6 \quad \Rightarrow \quad n = \frac{600}{5} = 120 \text{ Hz}$

The frequency of A

$$n_A = \left(n + \frac{2}{100}n\right) = 120 + \frac{2}{100} \times 120$$

$$= 122.4 \text{ Hz}$$

7. **(a, c)** $v_{\max} = a\omega = \frac{v}{10} = \frac{10}{10} = 1 \text{ m/s}$

$$\Rightarrow \quad a\omega = a \times 2\pi n = 1$$

$$\Rightarrow \quad n = \frac{10^3}{2\pi} \qquad (\because a = 10^{-3} \text{ m})$$

Since $v = n\lambda \Rightarrow \lambda = \frac{v}{n} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2} \text{ m}$

8. **(b)** $n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow n_1 l_1 = n_2 l_2 = n_3 l_3 = k$

$$l_1 + l_2 + l_3 = l \Rightarrow \frac{k}{n_1} + \frac{k}{n_2} + \frac{k}{n_3} = \frac{k}{n}$$

$$\Rightarrow \quad \frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \cdots$$

9. **(d)** $<v> = \frac{v_1 + v_2}{2} = \frac{\alpha\sqrt{T_1} + \alpha\sqrt{T_2}}{2}$

$\Rightarrow$ Time taken $= \frac{2l}{\alpha(\sqrt{T_1} + \sqrt{T_2})}$

Alternate Solution:

$$\frac{dx}{dt} = V = \alpha\sqrt{T_1 + \left(\frac{T_2 - T_1}{l}\right)x}$$

$$\int_{x=0}^{x=l} \frac{dx}{\sqrt{T_1 + \left(\frac{T_2 - T_1}{l}\right)x}} = \int_0^t \alpha dt$$

on solving we get $t = \frac{2l}{\alpha(\sqrt{T_1} + \sqrt{T_2})}$

10. **(c)** At $t = 0$, $y = 10 \sin 2\pi\left(\frac{50x}{22}\right)$

Change in pressure will be maximum at $y = 0$

$y = 0$ at $\frac{(2\pi)(50x)}{22} = 0, \pi, 2\pi, 3\pi, \ldots 100x\pi$

$= (3\pi)(22)$

or $x = 0.66$ m

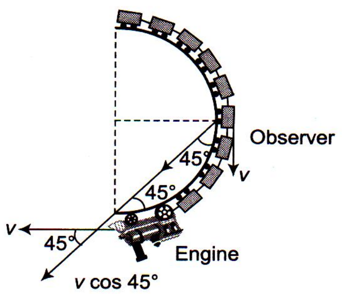
11. **(c)** Force closed pipe, $f = \frac{nV}{4\ell}$, $n = 1, 3, 5...$

$$f_1 = \frac{V}{4\ell} = \frac{330}{(4)(93.75/100)} = 88 \text{ Hz}$$

$$f_2 = \frac{3V}{4\ell} = \frac{(3)(330)}{(4)(93.75/100)} = 264 \text{ Hz}$$

Required $f = 264$ Hz

12. **(c)** The situation is shown in the fig. Both the source (engine) and the observer (Person in the middle of the train) have the same speed, but their direction of motion is right angles to each other. The component of velocity of observer towards source is $v \cos 45°$ and that of source along the time joining the observer and source is also $v \cos 45°$. There is number relative motion between them, so there is no change in frequency heard. So frequency heard is 200 Hz.



13. (a) When the train is approaching the stationary observer frequency heard by the observer $n' = \frac{v + v_0}{v} n$

when the train is moving away from the observer then frequency heard by the observer $n'' = \frac{v - v_0}{v} n$

it is clear that n' and n'' are constant and independent of time. Also and $n' > n''$.

14. (b) Equation of *A*, *B*, *C* and *D* are

$y_A = A \sin \omega t$, $y_B = A \sin(\omega t + \pi / 2)$

$y_C = A \sin(\omega t - \pi / 2)$, $y_D = A \sin(\omega t - \pi)$

It is clear that wave *C* lags behind by a phase angle of $\pi / 2$ and the wave *B* is ahead by a phase angle at $\pi / 2$.

15. (c) The particle velocity is maximum at *B* and is given by $\frac{dy}{dt} = (v_p)_{\max} = \omega A$

Also wave velocity is $\frac{dx}{dt} = v = \frac{\omega}{k}$

So slope $\frac{dy}{dx} = \frac{(v_p)_{\max}}{v} = kA$

16. (d) Given equation $y = y_0 \sin(\omega t - \phi)$

at $t = 0$, $y = -y_0 \sin \phi$

this is the case with curve marked *D*.

17. (c) We know frequency $n = \frac{p}{2l} \sqrt{\frac{T}{\pi r^2 \rho}} \Rightarrow n \propto \frac{1}{\sqrt{\rho}}$

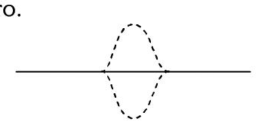
i.e., graph between *n* and $\sqrt{\rho}$ will be hyperbola.

18. (c) Energy density $(E) = \frac{I}{v} = 2\pi^2 \rho n^2 A^2$

$v_{\max} = \omega A = 2\pi n A \Rightarrow E \propto (v_{\max})^2$

i.e., graph between *E* and $v_{\max}$ will be a parabola symmetrical about *E* axis.

19. (c) After two seconds each wave travel a distance of 2.5 × 2 = 5 *cm* *i.e.* the two pulses will meet in mutually opposite phase and hence the amplitude of resultant will be zero.



20. (c) $n_Q = 341 \pm 3 = 344 Hz$ or $338 Hz$

on waxing *Q*, the number of beats decreases hence $n_Q = 344 Hz$

[MATHEMATICS]

41. (b) $\sin\theta+\cos\theta=1 \Rightarrow \frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta=\frac{1}{\sqrt{2}}$

Dividing by $\sqrt{1^2+1^2}=\sqrt{2}$,

we get $\sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}=\sin\frac{\pi}{4}$

$\Rightarrow \theta+\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{4}\Rightarrow \theta=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$.

42. (c) $\sin^2\theta=\frac{1}{4}=\sin^2\frac{\pi}{6}\Rightarrow\theta=n\pi\pm\frac{\pi}{6}$.

43. (b) $\cos^2\theta=\frac{3}{4}=\cos^2\left(\frac{\pi}{6}\right)\Rightarrow\theta=n\pi\pm\frac{\pi}{6}$.

44. (d) $\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta=\frac{\sqrt{2}}{2}$ {dividing by $\sqrt{(\sqrt{3})^2+1^2}=2$}

$\Rightarrow \sin\left(\theta+\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}=\sin\left(\frac{\pi}{4}\right)$

$\Rightarrow \theta=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{3}$.

45. (d) $\frac{1-\cos 2\theta}{1+\cos 2\theta}=3\Rightarrow\frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)}=3$

$\Rightarrow \tan^2\theta=3 \Rightarrow \theta=n\pi\pm\frac{\pi}{3}$.

46. (a) $\tan m\theta=\tan n\theta \Rightarrow m\theta=p\pi+n\theta\Rightarrow\theta=\frac{p\pi}{(m-n)}$

Hence different values of θ are in A.P. with $\frac{\pi}{m-n}$ as common difference.

47. (b) $2\sin A\cos^3 A-2\sin^3 A\cos A \quad =2\sin A\cos A(\cos^2 A-\sin^2 A)$

$=2\sin A\cos A\cos 2A=\sin 2A\cos 2A=\frac{1}{2}\sin 4A$.

48. (c) $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}$

$=\frac{\sin\theta+2\sin\theta\cos\theta}{2\cos^2\theta+\cos\theta}=\frac{\sin\theta(1+2\cos\theta)}{\cos\theta(1+2\cos\theta)}=\tan\theta$.

Trick : Put $\theta=30^\circ$, since for $\theta=30^\circ$ no option will give the common value.

49. (b) Given that $\sin\theta+\sin\phi=a$ .....(i)

and $\cos\theta+\cos\phi=b$ .....(ii)

Squaring, $\sin^2\theta+\sin^2\phi+2\sin\theta\sin\phi=a^2$

and $\cos^2\theta+\cos^2\phi+2\cos\theta\cos\phi=b^2$

Adding, $2+2(\sin\theta\sin\phi+\cos\theta\cos\phi)=a^2+b^2$

$\Rightarrow 2\cos(\theta-\phi)=a^2+b^2-2\Rightarrow \cos(\theta-\phi)=\frac{a^2+b^2-2}{2}$

$\Rightarrow \frac{1-\tan^2\frac{\theta-\phi}{2}}{1+\tan^2\frac{\theta-\phi}{2}}=\frac{a^2+b^2-2}{2}$

$\Rightarrow (a^2+b^2)+(a^2+b^2)\tan^2\frac{\theta-\phi}{2}-2-2\tan^2\frac{\theta-\phi}{2}$

$=2-2\tan^2\frac{\theta-\phi}{2}$

$\Rightarrow \frac{4-a^2-b^2}{a^2+b^2}=\tan^2\frac{\theta-\phi}{2}\Rightarrow \tan\frac{(\theta-\phi)}{2}=\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$

Trick : Put $\theta=\frac{\pi}{2},\phi=0^o$, then $a=1=b$

$\therefore \tan\frac{\theta-\phi}{2}=1$, which is given by (a) and (b).

Again putting $\theta=\frac{\pi}{4}=\phi$, we get $\tan\frac{\theta-\phi}{2}=0$, which is given by (b).

50. (b) $\cos A = \frac{3}{4} \Rightarrow \sin A = \frac{\sqrt{7}}{4}$

L.H.S $= 16(\sin 3A - \sin 2A)$

$= 16 \sin A(3 - 4\sin^2 A - 2\cos A)$

$= 16.\frac{\sqrt{7}}{4}\left(3 - 4.\frac{7}{16} - 2.\frac{3}{4}\right) = -\sqrt{7}$.

51. (d) $\frac{\cos A}{1-\sin A} = \frac{\cos A(1+\sin A)}{\cos^2 A} = \frac{(1+\sin A)}{\cos A}$

$$= \frac{\left(\cos\frac{A}{2} + \sin\frac{A}{2}\right)^2}{\left(\cos\frac{A}{2} + \sin\frac{A}{2}\right)\left(\cos\frac{A}{2} - \sin\frac{A}{2}\right)} = \frac{\cos\frac{A}{2} + \sin\frac{A}{2}}{\cos\frac{A}{2} - \sin\frac{A}{2}}$$

$= \frac{1+\tan\frac{A}{2}}{1-\tan\frac{A}{2}}$, $\left(\text{Dividing } N^r \text{ and } D^r \text{ by } \cos\frac{A}{2}\right)$

$= \tan\left(\frac{\pi}{4} + \frac{A}{2}\right)$.

52. (a) $\cos(\alpha/2) = -\sqrt{\frac{1+\cos\alpha}{2}}$

$\cos\alpha = -\sqrt{1-\sin^2\alpha}$ [$\because \alpha$ lies in IIIrd Quadrant]

$= -\sqrt{1-\frac{9}{25}} = -\frac{4}{5}$ $\therefore\ \cos(\alpha/2) = -\sqrt{\frac{1-\frac{4}{5}}{2}} = -\frac{1}{\sqrt{10}}$.

53. (c) $\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\therefore \cos 3\theta = 4\frac{1}{2^3}\left(a + \frac{1}{a}\right)^3 - 3\frac{1}{2}\left(a + \frac{1}{a}\right)$

$\Rightarrow \cos 3\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)\left[\left(a + \frac{1}{a}\right)^2 - 3\right]$

$\Rightarrow \cos 3\theta = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$.

54. (d) $\sin A = \frac{4}{5} \Rightarrow \tan A = -\frac{4}{3}$, $(90^o < A < 180^o)$

$\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$, (Let $\tan\frac{A}{2} = P$)

$\Rightarrow -\frac{4}{3} = \frac{2P}{1-P^2} \Rightarrow 4P^2 - 6P - 4 = 0$

$\Rightarrow P = \frac{-1}{2}$ (impossible), hence $\tan\frac{A}{2} = 2$.

55. (c) $\sin 15^o = \sin(45^o - 30^o) = \frac{\sqrt{3}-1}{2\sqrt{2}} =$ irrational

$\cos 15^o = \cos(45^o - 30^o) = \frac{\sqrt{3}+1}{2\sqrt{2}} =$ irrational

$\therefore\ \sin 15^o \cos 15^o = \frac{1}{2}(2\sin 15^o \cos 15^o)$

$= \frac{1}{2}\sin 30^o = \frac{1}{2}.\frac{1}{2} = \frac{1}{4} =$ rational

$\therefore\ \sin 15^o \cos 75^o = \sin 15^o \sin 15^o = \sin^2 15^o$

$= \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{4-2\sqrt{3}}{8} =$ irrational

56. (a) Let $\frac{\sin 3\theta - \cos 3\theta}{\sin\theta + \cos\theta} = \frac{N}{D}$ (say)

Then $N = 3\sin\theta - 4\sin^3\theta - (4\cos^3\theta - 3\cos\theta)$

$= 3(\sin\theta + \cos\theta) - 4(\sin^3\theta + \cos^3\theta)$

$= (\sin\theta + \cos\theta)\{3 - 4(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)\}$

$\therefore \frac{N}{D} + 1 = \frac{(\sin\theta + \cos\theta)\{3 - 4(1 - \sin\theta\cos\theta)\}}{\sin\theta + \cos\theta} + 1$

$= 3 - 4(1 - \sin\theta\cos\theta) + 1 = 4\sin\theta\cos\theta = 2\sin 2\theta$.

57. (d) $a\cos\theta + b\sin\theta = \sqrt{a^2 + b^2}\left(\frac{a\cos\theta}{\sqrt{a^2 + b^2}} + \frac{b\sin\theta}{\sqrt{a^2 + b^2}}\right)$

$= \sqrt{a^2 + b^2}\sin(\theta + \phi)$

Since, $-1 < \sin(\theta + \phi) < 1$,

Then $-\sqrt{a^2 + b^2} < \sin(\theta + \phi) < \sqrt{a^2 + b^2}$.

58. (d) Let $f(\theta) = 5\sin^2\theta + 4\cos^2\theta = 4 + \sin^2\theta$

$\therefore f(\theta) \ge 4 + 0$ $(\because \sin^2\theta \ge 0)$

$\therefore$ The minimum value of $f(\theta)$ is 4.

59. (d) A.M. $\ge$ G.M.

$\Rightarrow \frac{9\tan^2\theta + 4\cot^2\theta}{2} \ge \sqrt{4\cot^2\theta . 9\tan^2\theta}$

$\Rightarrow 9\tan^2\theta + 4\cot^2\theta \ge 12$

Therefore, the minimum value is 12.

60. (d) Given that *ABCD* is a cyclic quadrilateral.

So $A + C = 180° \Rightarrow A = 180° - C$

$\Rightarrow \cos A = \cos(180° - C) = -\cos C$

$\Rightarrow \cos A + \cos C = 0$ .....(i)

Similarly, $\cos B + \cos D = 0$ .....(ii)

Adding, $\cos A + \cos B + \cos C + \cos D = 0$.